参赛队员姓名： 汪言爱、谭淇骏、曾禹皓

中学：重庆市巴蜀常春藤学校

省份：重庆

国家：中国

指导教师姓名：刘又铭

指导教师单位：重庆市巴蜀常春藤学校

论文题目：Tragedy of the Commons in Transportation

Tragedy of the Commons in Transportation

Yanai Wang, Qijun Tan, Yuhao Zeng

August 17, 2022

Abstract

In transportation, the problem of the tragedy of the commons manifests itself in the form of road congestion: the social optimum and individual optimum are not consistent with each other. We suggest a real-time charging method to handle this issue in accordance with the optimal congestion pricing scheme. Using taxi data from New York City, we optimize drivers' route selections under the proposed fee scheme and find that the overall journey time during peak hours for three random days in 2010 decreases by 7% to 19%.

Keywords: Tragedy of the Commons; Road Congestion; Real-Time Surcharge

Contents

1 Introduction

Severe traffic congestion has long been an exigent issue in growing large cities, thwarting drivers from executing their plans in punctuality, exacerbating air pollution as an effect of car fuel emissions, and handicapping people from productivity and efficiency. According to the 2018 INRIX global traffic scorecard, the typical American motorist loses 97 hours per year due to congestion, spending $87 billion. (INRIX, 2019). In addition to impeding drivers from efficiently following their itineraries, traffic congestion also incurs increased energy usage, more pollution, and higher incidents of traffic accidents. According to a mathematical model, in comparison to the range between 40kmph to 70kmph, the range of speed between 0 kmph to 40 kmph increases fuel consumption from 16.66 km/l to 12.5 km/l, indicating a reduction of 24.96% in fuel economy of the vehicles in traffic congestions (Jayasooriya & Bandara, 2017). Shown by the case of Ghana, the levels of heavy metals emitted from vehicles along roads in Accra, a road with high traffic are relatively high compared to other roads where traffic volume is low, which means that traffic congestion is highly associated with air pollution (Armah et al., 2010). Moreover, "increased traffic congestion coupled with unsafe driving behaviors are creating an increase in the number of automobile accident fatalities" (Bull Attorneys).

Presented with all these prevailing issues, therefore, governments have attempted numerous approaches to combating excessive road congestion in ways such as restricting traffic flow on certain streets, adding traffic lights, expanding road capacities, etc. However, each method proposed poses different economic challenges and accompanies inevitable limitations, such as engendering provisional imbalance, catalyzing resource-wasting, and, in particular for the method of increasing road capacities, leading to inverse consequences. Dietrich Braess, a German mathematician, quantitatively demonstrated in 1968 that building a new road might really increase travel time for everyone, contrary to what the general public believes (Braess, 1968). His discovery of this counterintuitive phenomenon has come to be known as the Braess's Paradox (BP), suggesting that a malfunctioning network could be ameliorated by removing parts of it. BP was subsequently able to explain many instances in which the closure of existing important roadways helped traffic flow, such as in 1990, when the shutting of 42nd Street in New York City for Earth Day alleviated congestion in the region. Hence, evidently, utilizing BP as a tool in public transportation planning allows planners to enhance the overall efficiencies of road networks.

1.1 Outline

In this study, we examine the existing data and provide an analytical solution to the tragedy of the commons dilemma. To obtain the social optimum, we offer a real-time pricing strategy based on the analytical solution. We conclude that the fee should match the cost of externalities associated with adding a marginal car to the road. We employ a so-called "least-iteration-cycle" strategy to improve taxi drivers' route selections under the proposed fee scheme by combining New York taxi data with smaller data sets. We discover that the overall travel time decreases by 7% to 19% during peak hours on three random days in 2010, demonstrating that the fee system may greatly relieve the congestion situation.

1.2 Literature Review

Congestion pricing, particularly when applied to traffic congestion, is not a new concept in public policy. Pigou was most likely the first person to propose that road users should be taxed according to their marginal external costs. Knight also addressed this subject over a century ago (Pigou, 1912). The congestion pricing of marginal external costs has remained the primary premise in transportation economics and is referred to be the optimal congestion control technique (Phang & Toh, 2004). However, the first-best pricing scheme is impractical when sufficient data is available, hence the second-best pricing schemes have been extensively addressed in the literature (Verhoef, 2005). In this work, we are able to calculate the first-best strategy for the first time in practice using current technology.

In 1975, Singapore's governmental policies included the implementation of congestion charging. The government created a restricted driving area from 7:30 a.m. to 9:30 a.m. during peak traffic hours and imposed additional tolls. This limitation decreased peak-hour traffic in the restricted zone by 45 percent. After the implementation of the traffic estimating and prediction tool (TrEPS) in 2010, the restricted area's traffic volume decreased by an additional 10 to 30 percent (Channel New Asia, 2012). A comparable congestion pricing scheme was implemented in London, United Kingdom in 2003, using automated license plate recognition from 7:00 am to 6:00 pm, Monday through Friday. The city's public transport usage climbed by 50 to 60 percent, while the number of autos accessing the core area declined by 21 percent. In addition to reducing environmental pollution and the number of accidents, the collected income may be utilized to promote public benefits. Currently, Australia, Singapore, Austria, Finland, Germany, Italy, Malta, Norway, Sweden, United Kingdom, United Arab Emirates, Brazil, and Chile all have some form of a congestion road pricing scheme, and the results indicate that peak hour travel demand is consistently reduced, along with other social and environmental negative externalities such as air pollution, greenhouse gas emissions, visual intrusion, noise, and road accidents. All of these solutions are successful primarily by reducing demand, therefore they cannot address the tragedy of the commons dilemma outlined in the basic model (Figure 1).

1.3 Traffic Regulation Based on Economic Principles

As shown in Figure 1, the traffic flow in the traffic problem should be determined by the equilibrium point bounded jointly by the demand and cost curves. In the absence of a surcharge, travelers do not pay for the cost of congestion they impose on others, but they do pay for the cost of congestion others impose on them. In other words, they pay the average cost of travel (AC), not the marginal cost (MC).

Since the social cost of travelers' trips is the marginal cost, but the actual cost paid is the average cost, what is finally reached is an inefficient equilibrium point, i.e., the intersection of the AC curve and the demand curve in Figure 1, point b, corresponding to $V_{equilibrium}$ (V_e). Ideally, the marginal cost curve and the demand curve should reach an optimal equilibrium at point h and correspond to a smaller traffic volume $V_{optimal}$ (V_o). Therefore, the traffic congestion caused by deadweight loss is marked by the yellow striped triangular shaded area *hcb*.

The social planners can minimize the deadweight loss by modifying the average cost of travelers by means of a surcharge, which will cause the average cost curve to rise, as shown in the Figure 2. Since marginal cost is the first-order derivative of total cost, the surcharge as a constant term will be dropped, so the marginal cost curve is not affected by the surcharge. When the surcharge is τ, the modified average cost intersects the demand curve at point h and reaches the optimal traffic volume V_o.

1.4 Assumptions

1. **Perfect Information**
   Assumption: All drivers have perfect information for their route decision.
   Justification: Advanced traveler information systems (ATIS) such as Google Maps and Apple Maps are available to drivers for free and a perfect real-time information.

2. **Rational Drivers**
   Assumption: It is believed that drivers would choose the route with the shortest travel time or lowest cost depending on available information.
   Justification: Most drivers commuting on the roads tend to arrive at their destinations efficiently and therefore do not have preferences of routes.

3. **Homogeneous Vehicles**
   Assumption: All vehicles the road are identical.
   Justification: Although vehicles differ in size, speed, drivers' . For the sake of simplicity, this variation in driving patterns will be disregarded.

4. **Unchangeable Route**
   Assumption: Once a driver decides a route from the starting point to the destination, the route will not be changed.
   Justification: For the purpose of traffic control, many road systems adopt directional lanes signaled by "double white" lines to restrict vehicles from switching lanes. Therefore, it is reasonable to assume that drivers cannot change routes once they enter some directional lanes.

1.5 Variable Definitions

A table of variables used in the study is given below.

Notation	Definition
x	Traffic Volume
T	Travel Time
S	Surcharge
m	Iteration Number

Table 1: Notations

2 The Model

2.1 The social planner's optimization problem

Consider all cars to be identical (no size difference). The road system consists of N connections, with i representing the ith link. The value x indicates the traffic flow (number of automobiles that pass by in a given period) (We use traffic flow and traffic volume interchangeably in this paper).

Assume that drivers might have diverse origins and destinations (ODs) and that the social planner is aware of all ODs. The objective of the social planner is to minimize the total amount of travel time T:

$$\min_{x_i} \sum_i x_i T(x_i) \quad \text{for } i = 1, 2....N \tag{1}$$

subject to: ODs are fixed.

Note that the ODs are fixed places a limitation on the social planner. In other words, this model does not let OD demand to adjust to governmental policy. We will discuss more about this assumption later.

2.2 The social cost

We suggest using a real-time charge mechanism to optimize the issue of social planners. The fee structure will ensure that all societal expenses are absorbed. Let's first examine how much are the societal expenses.

Given the objective function of the social planner in Equation 1, if one marginal driver enters the ith link, the marginal cost is:

$$T(x_i) + x_i \frac{dT}{dx_i} \tag{2}$$

However, when the marginal driver attempts to choose a route, his/her private cost is just $T(x_i)$. Using a map application on the mobile device, he/she will make a decision based on the projected journey time for several routes. The only piece of information he/she will examine is the length of journey time given the existing circumstances. What he/she does not consider is that, once he/she joins the system, the additional car will increase traffic flow and alter the trip time for all other vehicles. $x_i \frac{dT}{dx_i}$ represents the externality effect/social cost, which is the difference between marginal total cost and marginal private cost.

To internalize the social cost, we implement a charge mechanism that requires the marginal motorist to pay $x_i \frac{dT}{dx_i}$ upon entering the link. We refer to the fee/charge as S, thus

$$S_{it} = x_i \frac{dT}{dx_i} \tag{3}$$

If x_i is a close to zero number, meaning there is little traffic flow, the charge is close to zero. If the marginal effect on the travel time, $\frac{dT}{dx_i}$, is zero, meaning the marginal vehicle has a zero externality effect, the charge is equal to zero.

2.3 The optimization strategy and computation method

Due to the OD limitation on the x traffic flow, it is impossible to develop an analytical solution to the optimization issue. We shall instead employ an iterative calculation approach.

For the OD pair of a mediocre driver, there are several possible routes from origin to destination. Use j to represent the jth route. The driver's purpose is to choose a route that minimizes travel time:

$$\min_j \sum_i I_{ji} T(x_i) \tag{4}$$

In this equation, Iji is an indicator equal to 1 if the i^{th} route is included in the j^{th} path, and 0 otherwise. In accordance with the pricing system, the purpose:

$$\min_j \sum_i I_{ji} [T(x_i) + x_i \frac{dT}{dx_i}] \tag{5}$$

If all individual drivers choose his/her optimal paths after taking social cost into consideration, the social optimum will be reached. The intuition of our computation method is to adjust all drivers' paths one by one and make enough iterations to eventually reach the social optimum.

The general optimization computation steps are outlined below. In the implications section, a more concrete model will be presented:

Start with the first newly arriving driver. Calculate the surcharge using Equation 3; given the trajectories of all other drivers, adjust to the ideal path using Equation 3. This phase requires the identification of all feasible routes from origin to destination, followed by the selection of the ideal route.

Step 2: Proceed to the next driver. Given the routes of all other drivers, including the modified one, repeat Step 1. When every driver has modified his or her route, one iteration is complete;

Step 3:Update the journey time for the connection based on the new traffic assignment; calculate the total travel time using the equation 1.

Step 4: Return to the first driver, update the surcharge using a "least-iteration-cycle" algorithm;

Step 5: Repeat the procedure until the system is stable (the difference in surcharge between iterations is less than or equal to 10^{-2}.

"least-iteration-cycle" is a heuristic receptive solution approach for guarantee the dynamic system converges to equilibrium. By decreasing the weight of the newly calculated surcharge and increasing the weight on the previous iteration, this repeated process guaranteed a equilibrium converges at the end, which is similar to MSA (the Method of Successive Average), a widely used algorithm for solving the traffic assignment.

$$S_{it} = \frac{1}{m} S_{at} + (1 - \frac{1}{m}) S_{a(t-1)} \tag{6}$$

Which m is the iteration number and a indicates average. The mathematics prove of system optimal solution holds at this converges point and the unique solution of this dynamic system please refer to the attachment.

3 Braess's Paradox Network Demonstration

The method described above will first apply to the famous Bracess's Paradox transportation network(Figure 2b) which is a 5-link 3 possible paths carrying 4000 vehicles from A to B. The original Bracess's paradox transportation network problem describe a road network that adding one or more will slow down overall traffic flow. As shown in Figure 2 (a), the original 4-links 2-path road network evenly distributes the road traffic due to the symmetric travel time functions. Therefore the drivers' individual travel time is 65 UOT (unite of time) and the total system travel time is 160,000 UOT. However, after adding the "0" cost link CD to the road network Figure 2 (b), the total system travel time has counter-intuitively increased to 320,000 UOT and individual travel time has also increased to 80 UOT, which is an increase of 23% compare with the original network. The reason behind this is too many drivers are attracted by this new route ACDB which leads to the congestion in some links of this road network. At the same time, other drivers who maintain the original route suffer a travel time increase, and force them to change to the new route, which caused these congested links has become more congested.

In the past, solving this Bracess's paradox problem was to identify and eliminate the redundant links. However, we now can apply the surcharge mechanism as another way to tackle this paradox. As the table 1 shows, with the 22.5 UOT surcharge on link AC and DB, the excessive congestion is eliminated. Only 500 drivers choice the new route ACDB and the total system time is reduced to 258,000, even 7.9% less than the original network. This result is exactly the mathematical system optimal solution through the KKT condition. Although the drivers choice different route travel time are various, with the surcharge they share the same travel cost. The detail of this heuristic approach is list below.

As the table 1 demonstrated, at the first iteration, due to the over congested traffic at links AC and DB, these two links charged at high surcharge rate 40 UOT which push all the traffic off the new route ACDB. At the second iteration, although all the traffic removed from the route ACDB, due to the "least-iteration-cycle" the surcharge rate only reduce to 30 UOT, the route ACDB is till too expensive to attract the drivers to choose this route. This situation was alleviated until the 5th iteration, where the surcharge rate reduced to 24 UOT. This surcharge rate decreasing decline trend continues and converged at the 28th iteration, which is 22.5 UOT surcharge on link AC and DB.

4 The Data

In this experiment, four datasets were used. The first is New York City speed and volume data, the second is New York City taxi data, including pickup and drop-off locations for each cab, the third is New York City road network structure data, and the fourth is New York City hourly link travel time. In the following section, we will discuss each dataset in further depth.

Multiple stations administered by the New York State Department of Transportation gather data on New York City's speed and traffic volume. The statistics are public knowledge and may be obtained from this link[1]. We use Region 11 speed data from 2014 to 2016 We additionally spatially match the data with the New York City taxi data and use the taxi data's station locations. The speed and volume statistics offer information on the number of vehicles detected within 15 distinct speed bands. The first speed range is between 0 and 20 miles per hour, the second between 20 and 25 miles per hour, etc. The top speed exceeds 85 miles per hour. Since

[1]https://www.dot.ny.gov/divisions/engineering/technical-services/highway-data-services/hdsb

region 11 is a metropolitan area, the majority of vehicles operate in the lower speed range. The figure depicts the dispersion of average speed (weighted by traffic volume). The average speed is 22.08 miles per hour.

We utilize the data on velocity and volume to determine the linear connection between velocity and density (Equation 8).

The New York City Cab & Limousine Commission (TLC) collects taxi trip data, which includes pick-up and drop-off locations, fare, payment method, passenger numbers, etc. Prior to the introduction of Green taxis in August 2013, Yellow taxis in New York City offered around 485,000 rides per day with an average distance of 2.6 miles. There are around 13,500 Yellow taxis and 30,000 taxi drivers in operation. Similar trends exist for pick-up and drop-off by time of day and day of the week. During the weekdays, there are two travel peaks that occur at 8 a.m. and 5 p.m. Therefore, we selected 8:00 am to 9:00 am and 5:00 pm to 6:00 pm for our model's test run.

OpenStreetMap (OSM), the primary data source for the road network structure in New York City, provides the following road network structure data: link nodes, GPS coordinates, direction, length, road type classifications, etc. GPS coordinates were used to map the New York taxi trip data to this data layer.

Although we have several speed and traffic data gathering stations, they do not cover every route segment. We thus utilize the anticipated traffic speeds derived from New York cab trip data (697,622,444 trips). The estimate was done by the research group led by Dan Work (DWG). Due to data record/storage problems, this estimate accounts for around 7.5% inaccuracies. The following table provides summary statistics on the data used in the implementation of the model for New York City. During peak hours, the average travel length is between 2.21 and 2.62 miles, and the average trip time is around 12 minutes (648.5   773.05 seconds).

5 Empirical Results

To estimate Equation 8, we do a linear regression with fixed effects on the speed and volume data for New York City. The outcome is shown in Table 4.

Then, we do iterations to determine the ideal real-time surcharge for achieving minimal overall trip time for the whole taxi system. In this procedure, a heuristic strategy, the ”least-iteration-cycle” method, is provided to identify all potential routes from a given origin to a particular destination (ODs). With the initial setting of each vehicle's OD in step 1, the least expensive route for each taxi is identified. Using the prior shortest route assignment from step 2, we then estimate the traffic flow and journey time. Next, the link marginal cost function is used to estimate the externality for the other cars on the connection if further vehicles use it. In order to limit the amount of back-and-forth, the Measure of Success Average (MSA) approach is used to step four. Each preceding iteration is accorded more significance than succeeding iterations. This method then returns to the first phase and continues until convergence conditions are met. Overall, the ”least-iteration-cycle” strategy looks for drivers' least expensive routes in each iteration, and each link's surcharge cost indirectly influences their route choices to internalize the impact of externality on all other drivers. The following information characterize the iteration:

Steps 1 through 3 are identical to Section ”The optimization strategy and computation method”.

Step 4: Update the surcharge to include 'least-iteration-cycle';

Step 5: Return the procedure until the convergence requirements are met:

$$\sum_i \frac{|S_{it} - S_{i(t-1)}|}{S_{it}} \leq 1\% \tag{7}$$

Using taxi trip data and transportation network information data from 2010, we apply this model to New York City. We randomly choose three weekdays (Jan. 5th, Feb. 2nd and October 5th, 2010) to determine the total amount of daylight saving time. We concentrate solely on the most busy hours, 8:00 am to 9:00 am (AM peak) and 5:00 pm to 6:00 pm (PM peak) (PM peak). The results are shown in Table 5. As the time value while determining the fee, we use \$20/h.

As shown in Table 5, the average cost per connection ranges from \$0.22 to \$0.92, and the entire system travel time improves by 7% to 19% depending on the date and peak hour. The model also predicts a 5–20% reduction in journey time. The average velocity rises by 27%, from 1%. According to the data on average connection time and average speed, the overall amount of time saved does not rely on the degree of congestion, but rather the traffic distribution during that time period. 2010 Jan. 5th, 2010 and Feb. 2nd, 2010 have comparable congestion levels (average link travel time around 163sec and average link speed around 1.35mph). However, while one network (Jan. 5th, 2010 5pm) could only improve 7 percent of total travel time, the other network (Feb. 2nd, 2010 8am) could save 17 percent.

Following the implementation of the fee, congestion is greatly decreased. Figure 6 illustrates the comparison of connection speed before and after the October 5th, 2010 fee. On the map, the network traffic speed diagram is separated into three categories: red indicates link speeds below 10 mph, yellow indicates speeds between 10 mph and 25 mph, and green indicates link speeds over 25 mph.

According to the speed diagrams, traffic conditions improve when the fee is implemented. The top left and bottom left panels display the link's average speed without a premium between 8:00 a.m. and 9:00 a.m. and 5:00 p.m. and 6:00 p.m. The panels on the top right and bottom right compare the speed map after the fee. It demonstrates that, after the application of the fee, the red color decreases and the yellow color grows, particularly in downtown corridors and main avenues. In New York City's downtown, avenues, which typically run perpendicularly from north to south, are often one or two lanes wider than streets, which run horizontally from east to west. The diagrams reveal that after the fee, vertical congestion on avenues is decreased more than horizontal congestion on streets. Due to quicker speeds and larger roads, when there is no fee an excessive number of cars prefer to congregate along avenues, causing congestion. Due to the increased price at crowded "avenues" when the surcharge is implemented, some traffic flow will divert to alternate routes, hence alleviating congestion.

Figure 7 and Figure 8 illustrate the taxi volume variations once the surcharge is implemented in further detail. The red line in the diagram depicts the decrease in taxi volume after the surcharge has been implemented, whereas the blue line reflects the rise in taxi volume. The width of the lines indicates the volume change's numerical value.

As shown in Figure Figure 7, the traffic volume on area 1 and area 2 decreases significantly at the 8am peak hour of October 5th, 2010. For instance, the FDR Drive serves to alleviate congestion in lower Manhattan. Because the FDR Drive is a three-lane roadway, it does not get crowded when 265 taxis transfer to this route from other routes. Nonetheless, as shown in Figure 6, these two regions continue to experience severe congestion despite the application of the premium.

The volume changes in regions 1 and 2 are different at 5 p.m. than they were at 8 a.m. on the same day. Due to the significant congestion in area 4, a portion of the traffic transfers to other routes in areas1 and 2 in an effort to alleviate the congestion. In addition, traffic on the Queens Midtown Tunnel and Ed Koch Queensboro Bridges transfers to other routes. As demonstrated in 8, the volume on these routes therefore rises.

Figure 9 and Figure 10 exhibit additional information on the surcharge rate, with the width of the lines denoting the surcharge rate's numerical value. The top panel of Figure 9 displays the surcharges for the road direction from west to east or north to south at 8:00 a.m. on October 5th 2010, while the bottom panel displays the surcharges for the road direction from east to west or south to north. It reveals that the greatest tolls are found on East-to-West bound bridges and tunnels that link Brooklyn to Manhattan during the morning rush hour. Moreover, owing to the high volume of traffic and the lack of alternate routes to John F. Kennedy International Airport (JFK), the tolls on the link highways in both directions are exorbitant.

Figure 10 depicts the surcharges at 5:00 p.m. on October 5th 2010 It demonstrates that, as a result of the traffic pattern from work to home, the biggest surcharges are placed at East-to-West and West-to-East bound bridges/tunnels, which vary from the morning peak hours.

6 Discussion and Conclusion

In this study, an analytical solution to the tragedy of the commons issue in traffic congestion is presented. To reach the societal optimum, we suggest a real-time charging system based on the analytical solution. Under the fee system, drivers would be penalized for choosing a more crowded route and will have an incentive to choose a less congested route. Then, all drivers' route selections are optimized for the whole transportation network. Due to the complexity of the transportation network, it is not possible to calculate numerically the ideal surcharge for the whole network system. Therefore, we use the ”least-iteration-cycle” method to reduce this computational load. By repeatedly simulating drivers’ route selection, we can determine the ideal real-time fee structure that dramatically lowers congestion. We apply the model to taxi data from New York City and show that the fee scheme can cut overall journey time by between 7% and 19% during peak hours on three random days in 2010.

Please note that since we only have information on taxi vehicles, the optimization is incomplete. We predict that once we have information on all automobiles, the system will improve even more. As autonomous cars are the future of transportation and several businesses are racing to develop these technologies, we can be assured that more transportation data will be accessible, and we will be able to adopt smart management in a more effective manner.

References

Braess, D. (1968). Über ein paradoxon aus der verkehrsplanung. *Unternehmensforschung*, *12*(1), 258–268.

INRIX. (2019). *Global trafic scorecard* (tech. rep.). INRIX Research.

Phang, S.-Y., & Toh, R. S. (2004). Road congestion pricing in singapore: 1975 to 2003. *Transportation Journal*, 16–25.

Pigou, A. C. (1912). *Wealth and welfare*. London, Macmillan; co., limited.

Verhoef, E. T. (2005). Second-best congestion pricing schemes in the monocentric city. *Journal of Urban Economics*, *58*(367-388).

Figure 1: Dead Weight Loss of Transportation

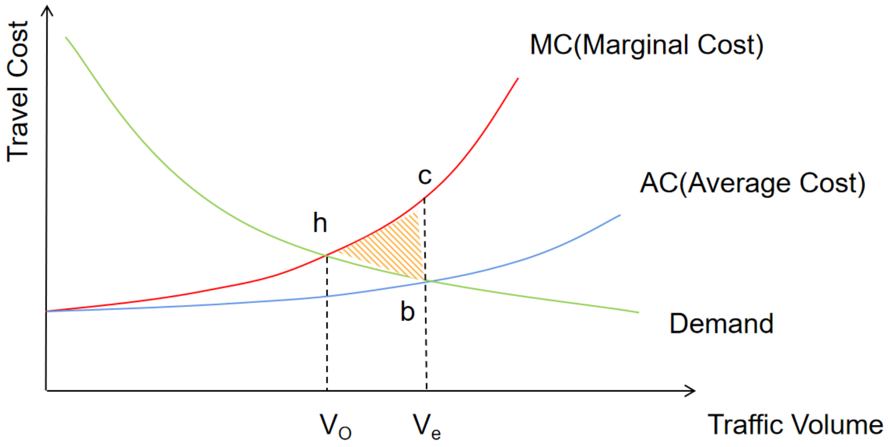


Figure 2: Correction of the Marginal Cost Curve

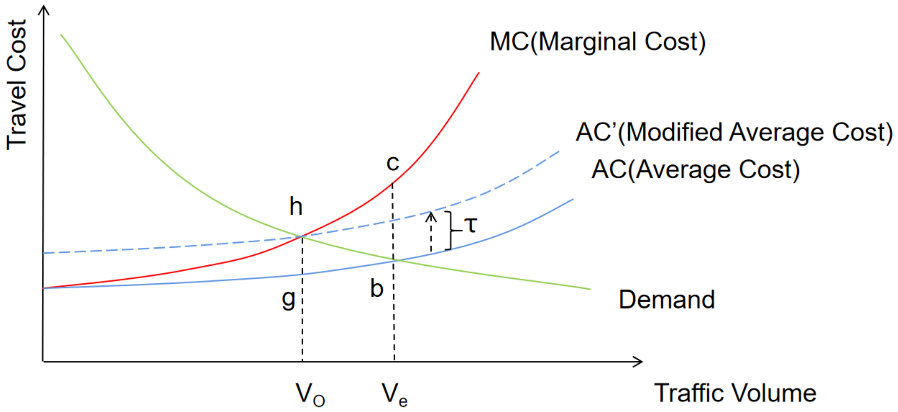


Figure 3: Social optimum for transportation flow

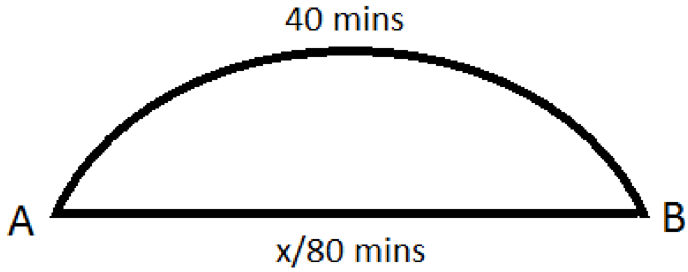


Figure 4: Braess's Paradox Network

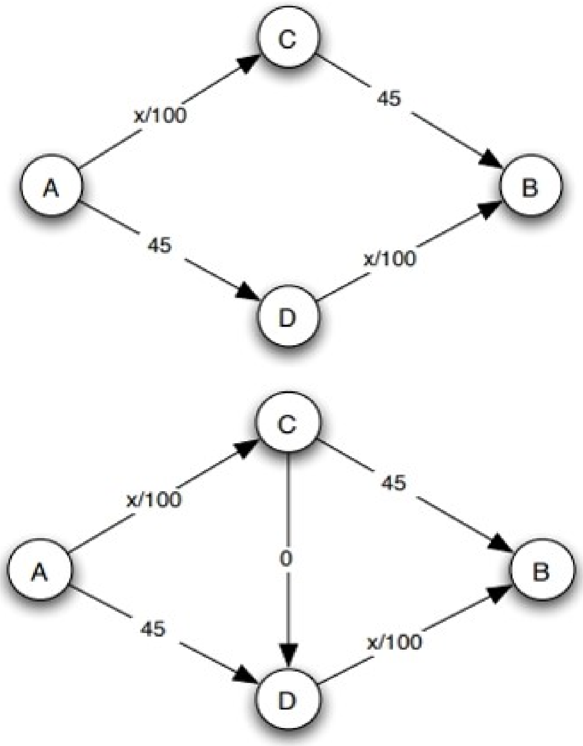


Table 2: Surcharge Rate Calculation Process

Initial							
Path	links	Surcharge rate	Path volume	Link Time cost	Path Travel cost	Total System Travel Time	Calculate Rate
A-C-D-B	AC	0	4,000	40	80	320,000	40
	CD	0		0			0
	DB	0		40			40
A-C-B	AC	0	0	40	85		40
	CB	0		45			0
A-D-B	AD	0	0	45	85		0
	DB	0		40			40
1st itertaion							
Path	links	New Surcharge	Path volume	Link Time cost	Path Travel cost	Total System Travel Time	Calculate Rate
A-C-D-B	AC	40	0	20	120	260,000	20
	CD	0		0			0
	DB	40		20			20
A-C-B	AC	40	2000	20	105		20
	CB	0		45			0
A-D-B	AD	0	2000	45	105		0
	DB	40		20			20
2nd itertaion							
Path	links	New Surcharge	Path volume	Link Time cost	Path Travel cost	Total System Travel Time	Calculate Rate
A-C-D-B	AC	30	0	20	100	260,000	20
	CD	0		0			0
	DB	30		20			20
A-C-B	AC	30	2000	20	95		20
	CB	0		45			0
A-D-B	AD	0	2000	45	95		0
	DB	30		20			20
3rd itertaion							
Path	links	New Surcharge	Path volume	Link Time cost	Path Travel cost	Total System Travel Time	Calculate Rate
A-C-D-B	AC	26.7	0	20	93.3	260,000.0	20.0
	CD	0.0		0			0.0
	DB	26.7		20			20.0
A-C-B	AC	26.7	2,000	20	91.7		20.0
	CB	0.0		45			0.0
A-D-B	AD	0.0	2,000	45	91.7		0.0
	DB	26.7		20			20.0
4th itertaion							
Path	links	New Surcharge	Path volume	Link Time cost	Path Travel cost	Total System Travel Time	Calculate Rate
A-C-D-B	AC	25	0	20	90	260,000	20
	CD	0		0			0
	DB	25		20			20
A-C-B	AC	25	2000	20	90		20
	CB	0		45			0
A-D-B	AD	0	2000	45	90		0
	DB	25		20			20
5th itertaion							
Path	links	New Surcharge	Path volume	Link Time cost	Path Travel cost	Total System Travel Time	Calculate Rate
A-C-D-B	AC	24	200	21	90	259,200	21
	CD	0		0			0
	DB	24		21			21
A-C-B	AC	24	1900	21	90		21
	CB	0		45			0
A-D-B	AD	0	1900	45	90		0
	DB	24		21			21
......							
20th itertaion							
Path	links	New Surcharge	Path volume	Link Time cost	Path Travel cost	Total System Travel Time	Calculate Rate
A-C-D-B	AC	22.4957407	500	22.5	90	258,750	22.5
	CD	0		0			0
	DB	22.4957407		22.5			22.5
A-C-B	AC	22.4957407	1750	22.5	89.9957407		22.5
	CB	0		45			0
A-D-B	AD	0	1750	45	89.9957407		0
	DB	22.4957407		22.5			22.5

Figure 5: Distribution of average speed

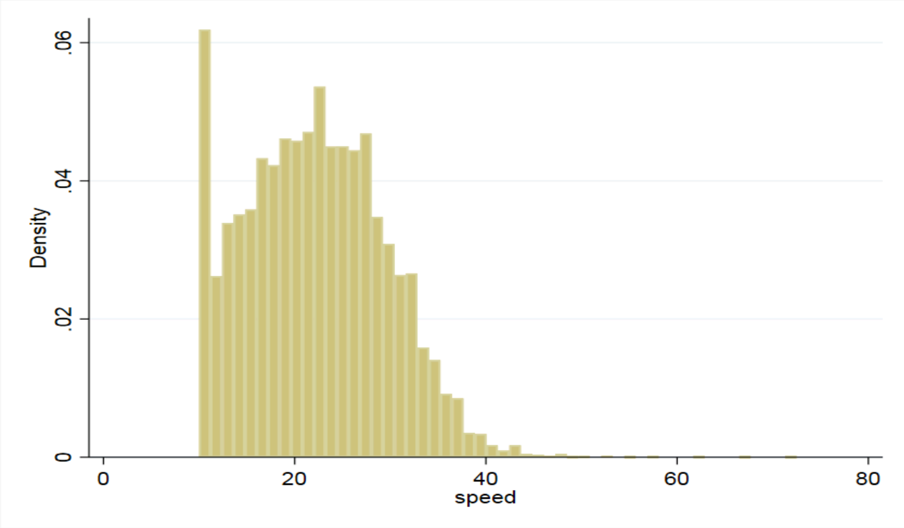
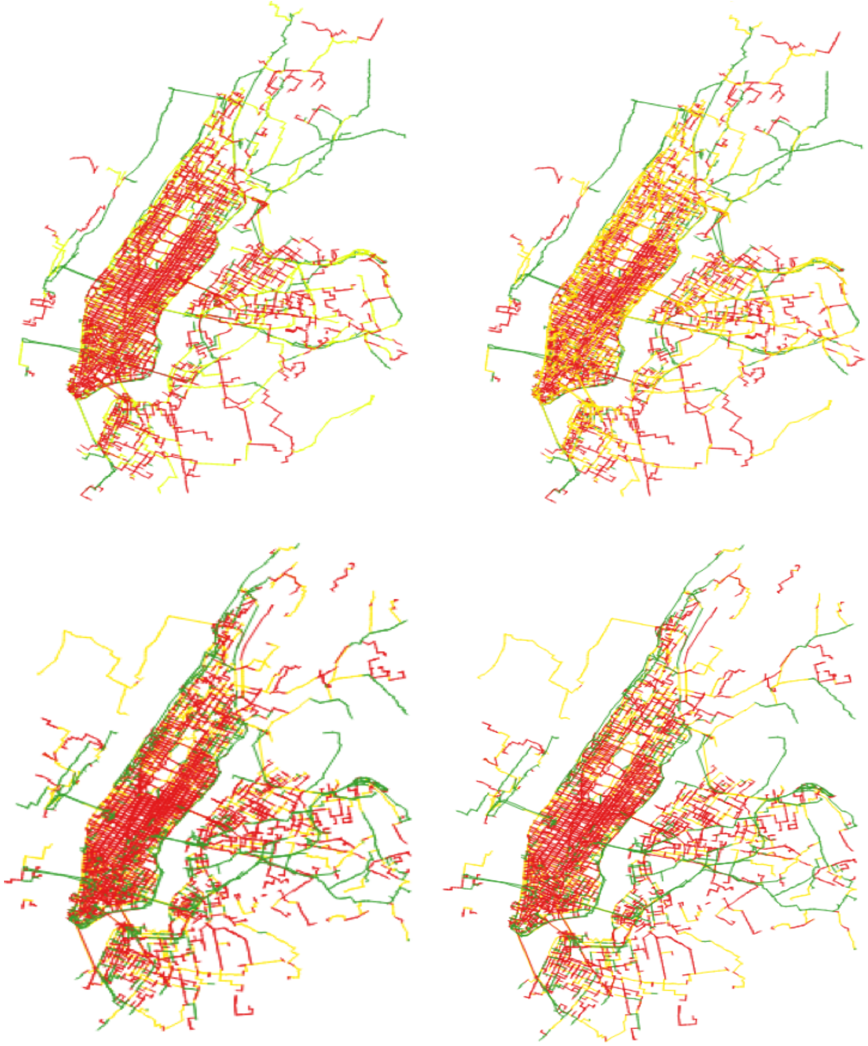


Figure 6: New York City peak hour speed diagram



Upper Left: Average traffic speed of 8:00am-9:00am without surcharge. Upper Right: Average traffic speed of 8:00am-9:00am with surcharge. Lower Left: Average traffic speed of 5:00pm-6:00pm without surcharge. Lower Right: Average traffic speed of 5:00pm-6:00pm with surcharge. All four panels are for October 5^{th}, 2010.

Figure 7: Taxi volume changes diagram for 8am

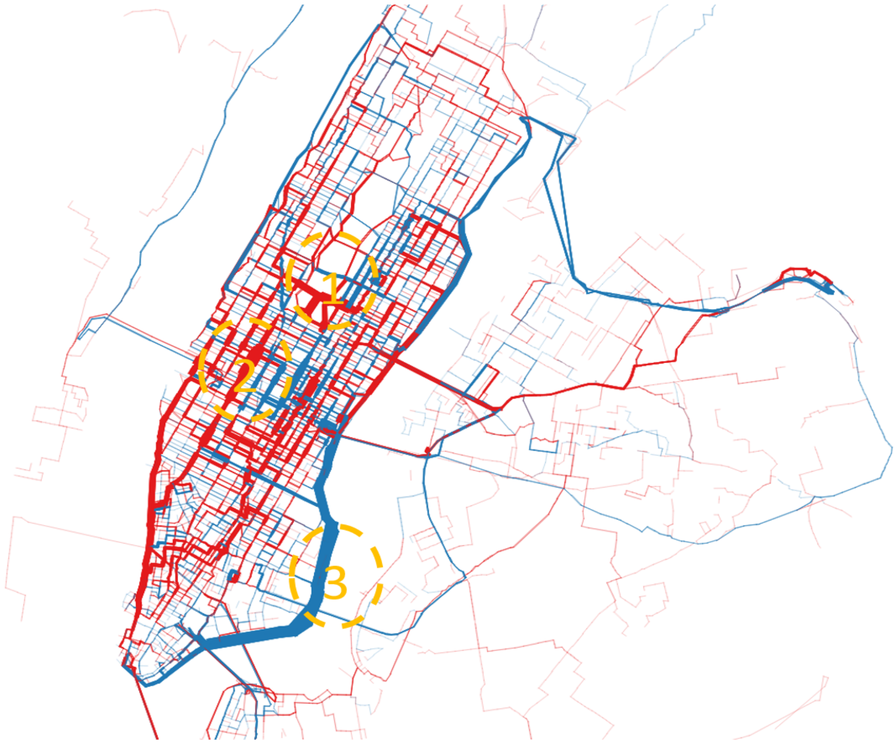


Figure 8: Taxi volume changes diagram for 5pm

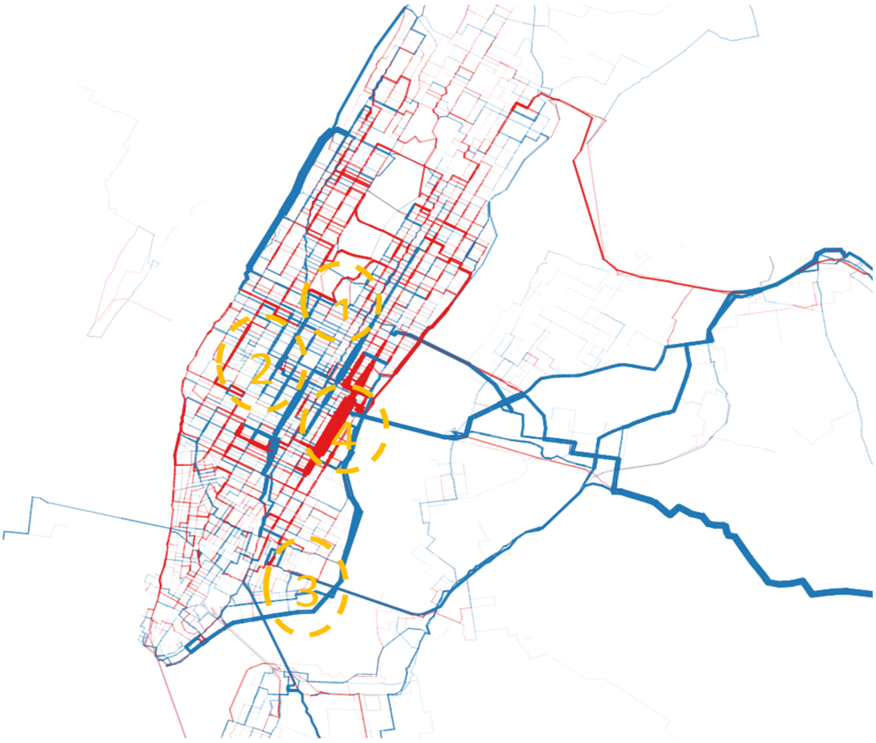
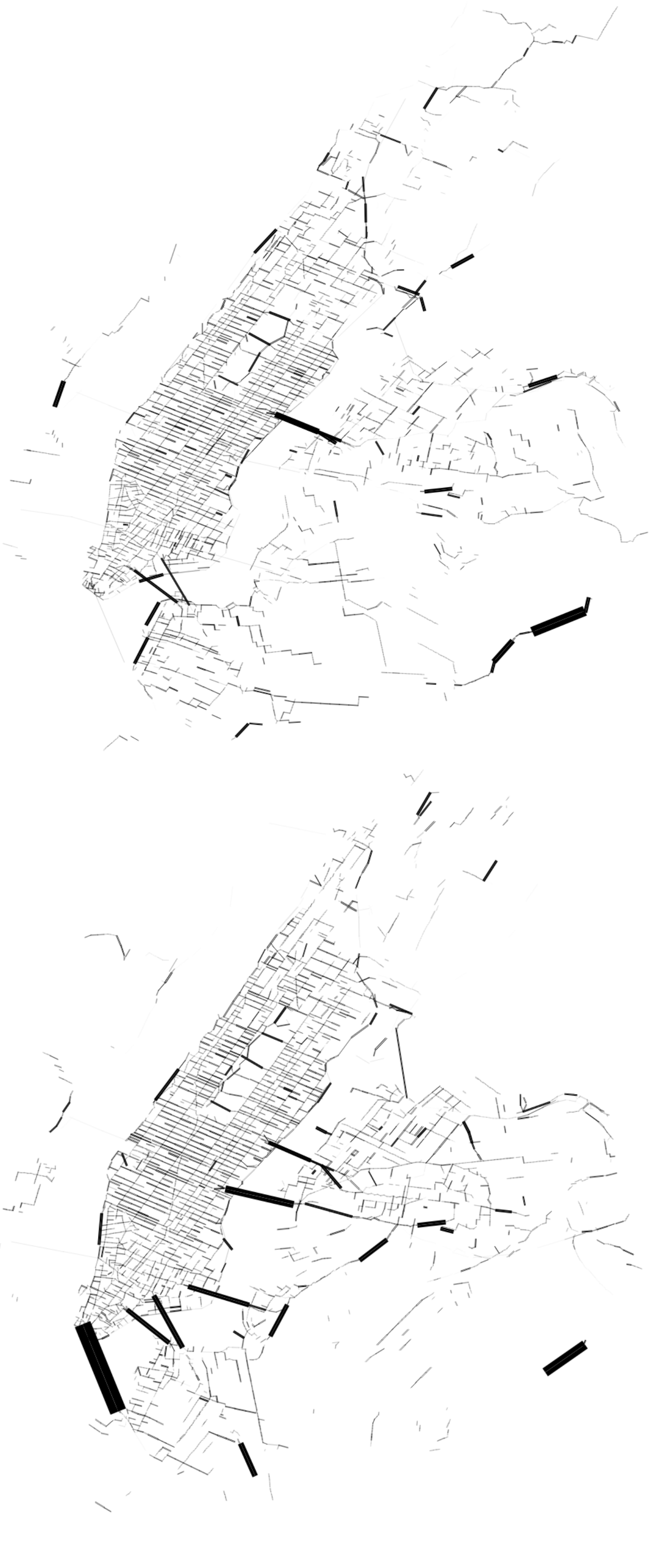
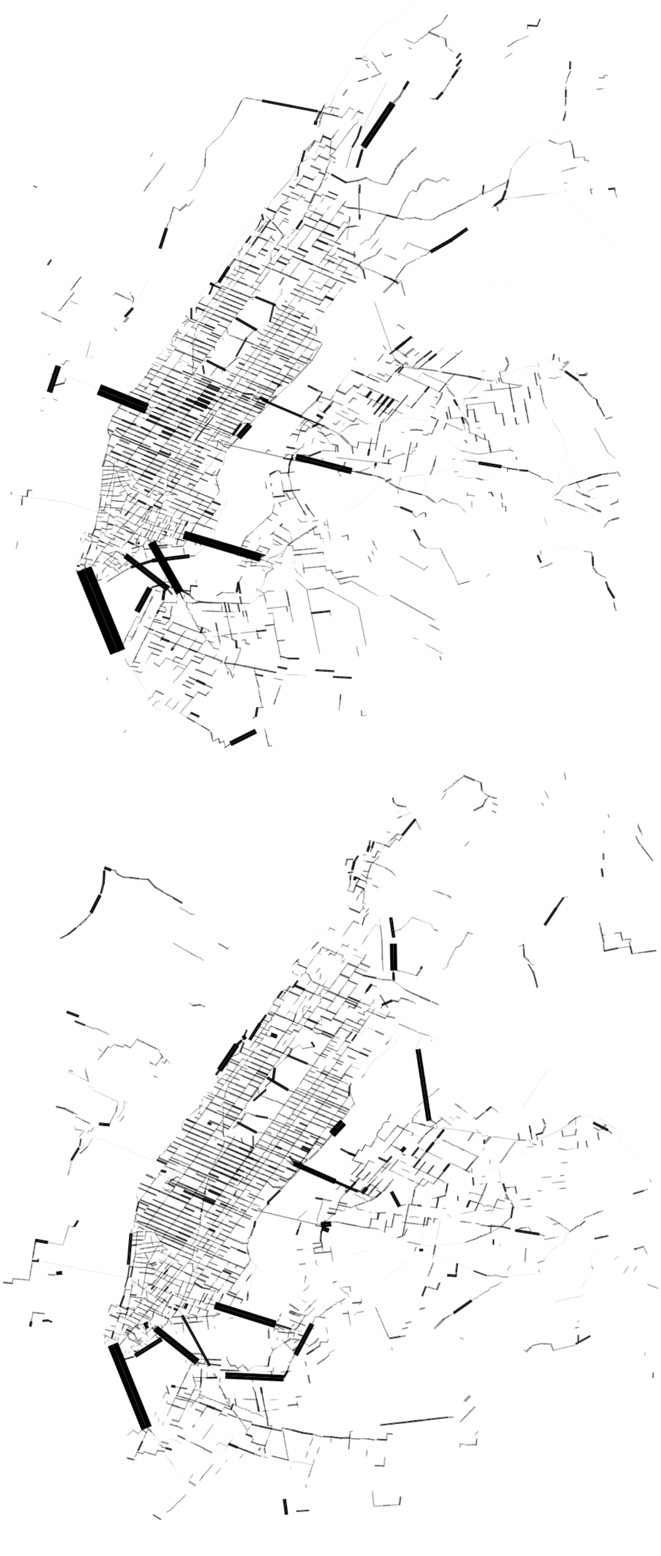


Figure 9: Surcharge rates at 8am



Top: West to East or North to South surcharge rate at 8am. Bottom: East to West or South to North surcharge rate at 8am. All panels are for October 5^{th}, 2010.

Figure 10: Surcharge rates at 5pm



Top: West to East or North to South surcharge rate at 5pm. Bottom: East to West or South to North surcharge rate at 5pm. All panels are for October 5th, 2010.

Table 3: Summary Statistics

Variables	Mean	Std. Dev.	Min	Max	Obs.	Dataset
Link Length (yard)	132.98	107.84	2.81	3937.12	260,855	OSM
Link travel Time (sec)	25.75	29.35	0.8	2781.11	84,480,169	DWG
Jan. 5th 8am trip length (mile)	2.32	2.79	0	33.9	29,858	TLC
Jan. 5th 5pm trip length (mile)	2.37	2.95	0	42.69	26,640	TLC
Feb. 2nd 8am trip length (mile)	2.21	2.43	0	33.6	13,678	TLC
Feb. 2nd 5pm trip length (mile)	2.26	3.02	0	172.6	11,542	TLC
Oct. 5th 8am trip length (mile)	2.42	2.83	0	31.3	27,913	TLC
Oct. 5th 5pm trip length (mile)	2.62	3.28	0	49.3	24,586	TLC
Jan. 5th 8am Original trip time (sec)	674.65	494.44	0	10641	29,858	TLC
Jan. 5th 5pm Original trip time (sec)	687.81	572.02	0	8220	26,640	TLC
Feb. 2nd 8am Original trip time (sec)	658.43	444.5	0	4425	13,678	TLC
Feb. 2nd 5pm Original trip time (sec)	648.5	457.33	1	6516	11,542	TLC
Oct. 5th 8am Original trip time (sec)	743.14	534.31	0	7939	27,913	TLC
Oct. 5th 5pm Original trip time (sec)	772.05	587.17	0	6007	24,586	TLC

Table 4: Speed and density

Variable	Coef.	Std. Err.	P> $\|t\|$
Constant	27.48	0.098	0.000
desinty	-0.24	0.001	0.000
Fixed effects of monitoring stations		Yes	
Observation number	199,165		
Adjusted R-square	0.74		

Table 5: Total time savings

Date	Time	Avg. link time (sec)		Average Speed (mph)		Avg Link surcharge	Total System Time(sec)		
		Before	After	Before	After	Rate	Before	After	Percent
1/5/2010	8am	76.54	63.43	14.49	15.84	$0.42	63,966,078	52,722,606	18%
	5pm	163.43	152.34	13.30	15.61	$0.92	121,932,803	113,241,243	7%
2/2/2010	8am	162.98	133.27	15.62	17.35	$0.81	68,224,757	56,362,618	17%
	5pm	125.97	100.97	14.77	16.17	$0.63	42,586,399	34,666,136	19%
10/5/2010	8am	26.48	24.85	8.69	9.68	$0.25	20,453,775	18,197,076	11%
	5pm	24.19	22.88	9.40	10.24	$0.22	18,231,036	16,568,184	9%

Figure 11: Function Forms

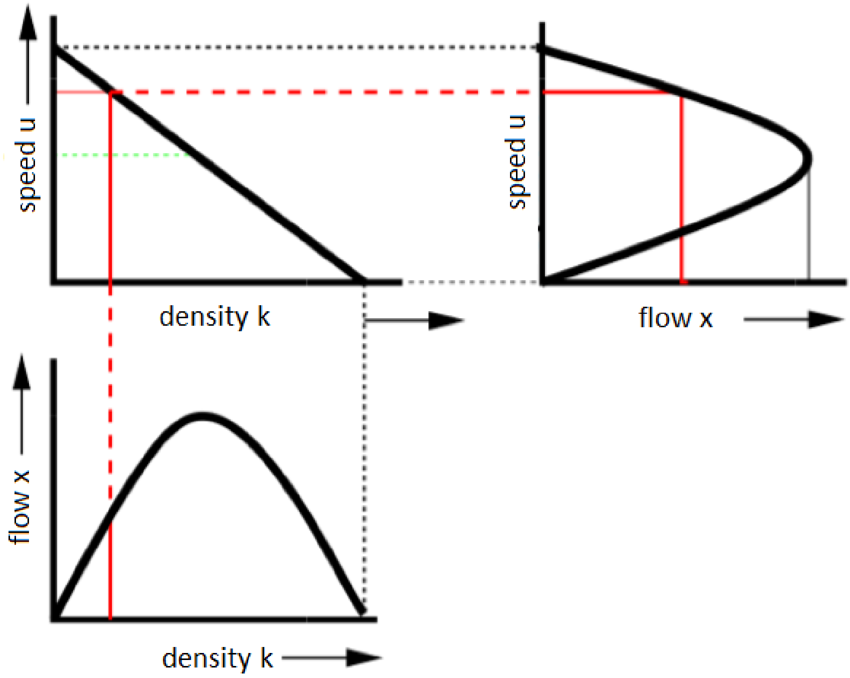
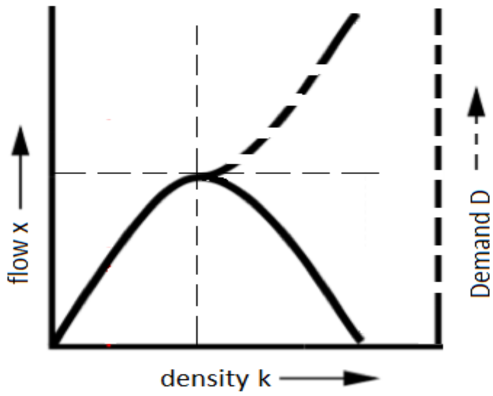


Figure 12: Traffic flow and demand



7 Appendix A:

To apply the model using empirical data, the function form of the journey time $t(x)$ must be specified. Less straightforward is the link between trip time and traffic flow in the transportation literature. Relationships between average speed, traffic flow, and traffic density are typical functions. Calculating trip time by dividing lane length by speed.

Empirical research from the transportation literature indicates that speed is a linear function of population density. The velocity is represented by u and the density by k. So:

$$u = \alpha_0 + \alpha_1 k \tag{8}$$

In which $\alpha_0 > 0$ and $\alpha_1 < 0$, and the road density is defined by:

$$k \equiv \frac{x}{u} \tag{9}$$

We may construct the functions between speed and traffic flow and between traffic flow and density by combining these two equations. The linear and nonlinear functions are shown in figure 11.

Note that the connection between speed u and traffic flow x is nonlinear. When the traffic flow is very low, there is either no traffic and the speed is very high, or there is traffic and the speed is very low. Traffic volume is also a nonlinear function of population density. There is either no traffic (high speed and low density) or traffic congestion when the traffic flow is minimal (low speed and high density).

If we add one mediocre vehicle to one connection, we anticipate that the speed will decrease. Based on Equation 8, the density will be greater. In other words, the rise in road demand is proportional to the density. If there is no traffic congestion, the demand for transportation is equivalent to the observed traffic flow. However, when congestion exists, the transportation demand exceeds the apparent traffic flow.

We assume, for the sake of simplicity, that the potential demand is proportional to the traffic flow after the road's maximum flow is achieved and congestion begins. If one marginal vehicle enters a connection, the potential demand increases by one vehicle, yet traffic flow may grow or decrease. This is seen in Figure 12. Marking D as $D*$ when x reaches its maximum allows us to express the link between D and x as:

$$x = \begin{cases} D, & \text{if } 0 < D \leq D^* \\ 2D^* - D, & \text{if } D > D^* \end{cases} \tag{10}$$

Given the relationship between traffic flow, density, speed and demand, we can finally write down the function for travel time. Use l to indicate one link's length, the travel time is:

$$T = \frac{l}{u} \tag{11}$$

According to Equation 9 and 10,

$$T = \frac{l}{u} = \frac{l}{x/k} = \begin{cases} \frac{l}{D/k}, & \text{if } 0 < D \leq D^* \\ \frac{l}{(2D^*-D)/k}, & \text{if } D > D^* \end{cases} \tag{12}$$

Correspondently, the surcharge can be written as a function of demand:

$$S = x\frac{dT}{dD} = \begin{cases} D\frac{dT}{dD}, & \text{if } 0 < D \leq D^* \\ (2D^* - D)\frac{dT}{dD}, & \text{if } D > D^* \end{cases} \tag{13}$$

8 Appendix B:

In the empirical section, we will estimate $alpha$ and $alpha1$ in Equation 8 and then compute the surcharge using the values presented in Appendix.

The empirical evidence shows that:

$$u = 27.48 - 0.24k$$

As $u = \frac{x}{k}$, k is a function of x as following:

$$k = \begin{cases} 57.25 - \sqrt{3277.56 - 4.17x}, & \text{if } k \leq 57.25; \\ 57.25 + \sqrt{3277.56 - 4.17x}, & \text{if } k > 57.25; \end{cases}$$

Here $0 \leq x \leq 786$. Although the volume of traffic cannot exceed 786, the theoretical demand may. According to Figure 12, k is an increasing function of demand D. When $k \leq 57.25$,

$$k = 57.25 - \sqrt{3277.56 - 4.17D} \text{ if } 0 < D \leq 786$$

When $k > 57.25$, demand will surpass 786. For simplicity, we assume that the prospective demand is symmetric with respect to the traffic flow curve $x = 786$. So when $k > 57.25$, $D = 1572 - x$, and:

$$k = 57.25 + \sqrt{-3277.68 + 4.17D} \text{ if } D > 786$$

Combining these two equations, the full function can be written as:

$$k = \begin{cases} 57.25 - \sqrt{3277.56 - 4.17D}, & \text{if } 0 < D \leq 786 \\ 57.25 + \sqrt{-3277.68 + 4.17D}, & \text{if } D > 786 \end{cases}$$

With the given the link length l, the link travel time function will be:

$$T = \frac{l}{u} = \frac{l}{x/k} = \begin{cases} \frac{l}{D/k}, & \text{if } 0 < D \leq 786 \\ \frac{l}{(1572-D)/k}, & \text{if } D > 786 \end{cases}$$

Therefore:

$$T = \begin{cases} \frac{l(57.25-\sqrt{3277.56-4.17D})}{D}, & \text{if } 0 < D \leq 786 \\ \frac{l(57.25+\sqrt{-3277.68+4.17D})}{1572-D}, & \text{if } D > 786 \end{cases}$$

The surcharge is written as:

$$S = \begin{cases} \frac{l(\frac{4.17}{2\sqrt{3277.56-4.17D}}-57.25+\sqrt{3277.56-4.17D})}{D}, & \text{if } 0 < D \leq 786 \\ \frac{l(\frac{4.17(1572-D)}{2\sqrt{-3277.56+4.17D}}+57.25+\sqrt{-3277.56+4.17D})}{(1572-D)}, & \text{if } D > 786 \end{cases}$$